Then $d^2y/dx^2=2c_2+6c_3x+12c_4x^2+20c_5x^3+30c_6x^4+$ etc. ...(2), and $axy=ac_0x+ac_1x^2+ac_2x^3+ac_3x^4+ac_4x^5+$ etc. ...(3).

Equating coefficients of like powers of x in (2) and (3), and reducing, we obtain by substituting in (1),

$$y=c_0\left(1+\frac{ax^3}{3!}+\frac{4a^2x^6}{6!}+\frac{4.7a^3x^9}{9!}+\frac{4.7.10a^4x^{12}}{12!}+\text{etc.}\right)$$

$$+c_1\left(x+\frac{2ax^4}{4!}+\frac{2.5a^2x^7}{7!}+\frac{2.5.8a^3x^{10}}{10!}+\frac{2.5.8.11a^4x^{13}}{13!}+\text{etc.}\right)$$

Also solved similarly by G. B. M. Zerr. Professor William Hoover did not give a solution but referred to the discussion of the general problem in Forsythe's *Differential Equations*, Chapter VII, page 217, Second Edition. A discussion of the solution of this class of differential equations, by definite integrals, is also given in Price's *Infinitesimal Calculus*, Vol. II., page 484.

MECHANICS.

187. **Proposed by M. E. GRABER, M. A., Heidelberg University, Tiffin, Ohio.**

Find the path described by a particle acted upon by a central force, the force being directly proportional to the distance of the particle.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

The equations of motion are $d^2x/dt^2+\mu x=0$ and $d^2y/dt^2+\mu y=0$.

$\therefore x=A\cos t\sqrt{\mu}+B\sin t\sqrt{\mu}$, and $y=C\cos t\sqrt{\mu}+D\sin t\sqrt{\mu}$.

$\therefore (Ay-Cx)^2+(By-Dx)^2=(AD-BC)^2$, an ellipse with center of force at center.

192 **Proposed by WILLIAM HOOVER, Ph D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.**

A solid sphere rolls down a trough formed by two planes which make with each other an angle 2α. Find, by the principle of *vis viva*, the expression for the time of rolling down the trough when the inclination of the trough to the horizon is β.

Solution by LEROY D. WELD, Coe College, Cedar Rapids, Iowa, and the PROPOSER.

Let C be the center of the sphere, O the center of the line joining the points of contact A and B of the trough and sphere, k the radius of gyration of the sphere about its center, r=its radius, θ=the angle through which a fixed radius in the plane of O and the edge of the angle 2ϕ, has rotated in any time t from the beginning of motion; let a plane and line, the first embracing AB, and the other passing through C, be drawn parallel to the edge of 2ϕ, both cutting a fixed horizontal plane, the line in the point D; x, y, the coördinates of C at the time t, D, the origin, and a and b the initial values of x, y; then by *vis viva*, m being the mass of the sphere,

$$m\left(\frac{dx^2}{dt^2}+\frac{dy^2}{dt^2}+k^2\frac{d\theta^2}{dt^2}\right)=C-2mgy.$$

When $y=b, \frac{dx}{dt}=0$, $\frac{dy}{dt}=0$, $\frac{d\theta}{dt}=0$; $\therefore C=2mgb$, and we have

$$\frac{dx^2}{dt^2}+\frac{dy^2}{dt^2}+k^2\frac{d\theta^2}{dt^2}=2g(b-y)\ldots(1).$$

Now $x=a-\theta r\sin\alpha\cos\beta$, $y=b-\theta r\sin\alpha\sin\beta$; then

$$\frac{dx}{dt}=-r\sin\alpha\cos\beta\frac{d\theta}{dt},\quad \frac{dy}{dt}=-r\sin\alpha\sin\beta\frac{d\theta}{dt},\quad \frac{dx^2+dy^2}{dt^2}=r^2\sin^2\alpha\frac{d\theta^2}{dt^2}; \text{ then}$$

$$k^2\frac{d\theta^2}{dt^2}=\frac{k^2}{r^2\sin^2\alpha}.\frac{dx^2+dy^2}{dt^2}.$$

Substituting in (1),

$$\frac{dx^2+dy^2}{dt^2}.\frac{r^2\sin^2\alpha+k^2}{r^2\sin^2\alpha}=2g(b-y)=2g\theta r\sin\alpha\sin\beta\ldots(2).$$

But $dx^2+dy^2=ds^2$, in which $s=\theta r\sin\alpha$, or $ds=r\sin\alpha d\theta$; $\therefore$ (2) becomes

$$\frac{ds^2}{dt^2}=\frac{2r^2\sin^2\alpha}{r^2\sin^2\alpha+k^2}gs\sin\beta\ldots(3).$$

Taking the derivative of both members of (3) with respect to t, dividing by $\frac{ds}{dt}$, multiplying by dt and integrating twice, noticing that when $t=0$, $\frac{ds}{dt}=0$,and $s=0$, and finally putting $s=l=$the length of the trough,

$$s=\tfrac{1}{2}\frac{r^2\sin^2\alpha}{r^2\sin^2\alpha+k^2}gt^2, \text{ whence } t=\frac{1}{\sin\alpha}\sqrt{\frac{(10\sin^2\alpha+4)l}{5g\sin\beta}},\ k^2 \text{ being } \tfrac{2}{5}r^2.$$

Also solved by G. B. M. Zerr and G. W. Greenwood.

DIOPHANTINE ANALYSIS.

136 Proposed by A H HOLMES, Brunswick, Maine.

In the equation in Diophantine Analysis: $2x^2+2x+1=\square=u^2$, show that u is always the sum of two squares.

Solution by L. E. NEWCOMB, Los Gatos, Cal.

$2x^2+2x+1=x^2+(x+1)^2\ldots(1)$.

Let $pq=x$ or $pq=x+1$ according as x is odd or even; then, for all integral values of x that satisfy (1), $\frac{1}{2}p^2-\frac{1}{2}q^2=x+1$ or $\frac{1}{2}p^2-\frac{1}{2}q^2=x$.

$\therefore p^2q^2+(\frac{1}{2}p^2-\frac{1}{2}q^2)^2=x^2+(x+1)^2=u^2$.

But $p^2q^2+(\frac{1}{2}p^2-\frac{1}{2}q^2)^2=(\frac{1}{2}p^2+\frac{1}{2}q^2)$. For p, substitute $m+n$, for q, $m-n$; then $(\frac{1}{2}p^2+\frac{1}{2}q^2)^2$ becomes $(m^2+n^2)^2$. Since $(m^2+n^2)^2=u^2$, $u=m^2+n^2$), the sum of two squares.